C3 Paper C – Marking Guide

1. $x(x-2) = 0$, $x = 0, 2$ $\therefore$ crosses x-axis at (0, 0) and (2, 0)

volume $= \pi \int_0^2 (x^2 - 2x)^2 \, dx$ — M1

$= \pi \int_0^2 (x^4 - 4x^3 + 4x^2) \, dx$

$= \pi[\frac{1}{5}x^5 - x^4 + \frac{4}{3}x^3]_0^2$ — M1 A1

$= \pi\{(\frac{32}{5} - 16 + \frac{32}{3}) - (0)\} = \frac{16}{15}\pi$ — M1 A1 **(5)**

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2. *(i)* $3x + 1 = e^2$ — M1

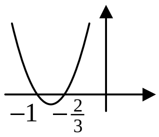
$x = \frac{1}{3}(e^2 - 1)$ — A1

(ii) consider $\ln(3x^2 + 5x + 3) \geq 0$

$\Rightarrow$ $3x^2 + 5x + 3 \geq 1$ — M1

$3x^2 + 5x + 2 \geq 0$

$(3x + 2)(x + 1) \geq 0$ — M1



$x \leq -1$ or $x \geq -\frac{2}{3}$ — A1

$\therefore$ if (e.g.) $x = -\frac{3}{4}$, $\ln(3x^2 + 5x + 3) = \ln\frac{15}{16} = -0.0645...$ — M1

$\therefore$ if $x = -\frac{3}{4}$, $\ln(3x^2 + 5x + 3) < 0$

$\therefore$ statement is false — A1 **(7)**

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3. *(i)* $= \frac{1}{3x-2} \times 3 = \frac{3}{3x-2}$ — M1 A1

(ii) $= \frac{2\times(1-x)-(2x+1)\times(-1)}{(1-x)^2} = \frac{3}{(1-x)^2}$ — M1 A2

(iii) $= \frac{3}{2}x^{\frac{1}{2}} \times e^{2x} + x^{\frac{3}{2}} \times 2e^{2x} = \frac{1}{2}x^{\frac{1}{2}}e^{2x}(3 + 4x)$ — M1 A2 **(8)**

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4. *(i)* $\cos^2 x = (\sqrt{3} - 1)^2 = 3 - 2\sqrt{3} + 1 = 4 - 2\sqrt{3}$ — M1

$\cos 2x = 2\cos^2 x - 1 = 2(4 - 2\sqrt{3}) - 1 = 7 - 4\sqrt{3}$ — M1 A1

(ii) $2(\cos y \cos 30 - \sin y \sin 30) = \sqrt{3}(\sin y \cos 30 - \cos y \sin 30)$ — M1 A1

$\sqrt{3}\cos y - \sin y = \frac{3}{2}\sin y - \frac{1}{2}\sqrt{3}\cos y$ — B1

$\frac{3}{2}\sqrt{3}\cos y = \frac{5}{2}\sin y$

$\tan y = \frac{3}{2}\sqrt{3} \div \frac{5}{2} = \frac{3}{5}\sqrt{3}$ — M1 A1 **(8)**

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5. *(i)* $f(x) = (x - \frac{3}{2})^2 - \frac{9}{4} + 7 = (x - \frac{3}{2})^2 + \frac{19}{4}$ — M1 A1

$\therefore$ $f(x) \geq \frac{19}{4}$ — A1

(ii) $= g(11) = 21$ — M1 A1

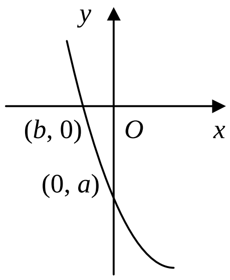
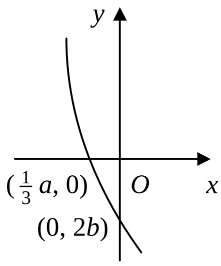
(iii) $fg(x) = f(2x - 1) = (2x - 1)^2 - 3(2x - 1) + 7$ — M1

$\therefore$ $4x^2 - 4x + 1 - 6x + 3 + 7 = 17$

$2x^2 - 5x - 3 = 0$ — A1

$(2x + 1)(x - 3) = 0$ — M1

$x = -\frac{1}{2}, 3$ — A1 **(9)**

6.	*(i)*	$4\sin x + 3\cos x = R\sin x\cos\alpha + R\cos x\sin\alpha$		
		$R\cos\alpha = 4,\ \ R\sin\alpha = 3$	M1	
		$\therefore\ \ R = \sqrt{4^2+3^2} = 5$	A1	
		$\tan\alpha = \frac{3}{4},\ \ \alpha = 0.644$ (3sf)	A1	
		$\therefore\ \ 4\sin x + 3\cos x = 5\sin(x + 0.644)$		
	(ii)	minimum = –5	B1	
		occurs when $x + 0.6435 = \frac{3\pi}{2},\ \ x = 4.07$ (3sf)	M1 A1	
	(iii)	$5\sin(2\theta + 0.6435) = 2$		
		$\sin(2\theta + 0.6435) = 0.4$	M1	
		$2\theta + 0.6435 = \pi - 0.4115,\ 2\pi + 0.4115$		
		$2\theta = 2.087,\ 6.051$	M1	
		$\theta = 1.04,\ 3.03$ (2dp)	A2	**(10)**
7.	*(a)*	*(i)* 	M1 A1	
		(ii) 	M1 A2	
	(b)	$x = 0\ \Rightarrow\ y = -1\ \ \therefore\ b = -1$	B1	
		$y = 0\ \Rightarrow\ 2 - \sqrt{x+9} = 0$		
		$x = 2^2 - 9 = -5\ \ \therefore\ a = -5$	M1 A1	
	(c)	$y = 2 - \sqrt{x+9},\ \ \sqrt{x+9} = 2 - y,\ \ \ x + 9 = (2-y)^2$		
		$x = (2-y)^2 - 9$	M1	
		$\therefore\ \ f^{-1}(x) = (2-x)^2 - 9$	A1	
		$f(-9) = 2\ \ \therefore$ domain of $f^{-1}(x)$ is $x \in \mathbb{R},\ x \le 2$	M1 A1	**(12)**
8.	*(i)*	$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} - 4e^{1-4x}$	M1 A1	
		grad = –3, grad of normal = $\frac{1}{3}$	M1	
		$\therefore\ \ y - \frac{3}{2} = \frac{1}{3}(x - \frac{1}{4})$ $\quad$ [$4x - 12y + 17 = 0$]	A1	
	(ii)	SP: $\frac{1}{2}x^{-\frac{1}{2}} - 4e^{1-4x} = 0$		
		$\frac{1}{2\sqrt{x}} = 4e^{1-4x}$		
		$\frac{1}{8\sqrt{x}} = e^{1-4x}$	M1	
		$8\sqrt{x} = e^{4x-1}$		
		$4x - 1 = \ln 8\sqrt{x}$	M1	
		$x = \frac{1}{4}(1 + \ln 8\sqrt{x})$	A1	
	(iii)	$x_1 = 0.7699,\ x_2 = 0.7372,\ x_3 = 0.7317,\ x_4 = 0.7308 = 0.731$ (3dp)	M1 A1	
	(iv)	let $f(x) = \frac{1}{2}x^{-\frac{1}{2}} - 4e^{1-4x}$		
		$f(0.7305) = -0.00025,\ \ f(0.7315) = 0.0017$	M1	
		sign change, $f(x)$ continuous $\therefore$ root	A1	
	(v)	$x_1 = 6.304,\ x_2 = 1.683 \times 10^{19}$		
		diverges rapidly away from root	B2	**(13)**

Total **(72)**